

# Unity of Fundamental Interactions

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The vector representation of the linearized gravitational field (the graviton field) or the so called quantum gravitodynamics which describes the motion of masses in a weak gravitational field is employed to understand the unity of the four known interactions. We propose a gauge group  $SU(3) \times SU(2) \times U(1) \times U(1)$  for such a unified field theory. In this paper we study the  $SU(2) \times U(1) \times U(1)$  sector of the theory and in analogy to the electroweak mixing angle we define a gravitoweak mixing angle. The unified gauge field theory predicts the existence of three massive vector bosons, the  $Y^\pm$  and the  $X^0$ , and two massless vector bosons, the photon and the graviton (in its vector representation). We determine the mass spectrum of the  $Y^\pm$  and the  $X^0$  and predict a modification to the fine structure constant under unified field conditions. Furthermore, we briefly discuss the implications of the extended object formulation for the gauge hierarchy problem.

## I. INTRODUCTION

The quantum mechanics of extended objects<sup>1</sup> and its infinite dimensional generalization, namely, the quantum field theory of extended objects, in particular  $\phi^6$  scalar field theory<sup>2</sup>, quantum electrodynamics with the Pauli term,<sup>3</sup> and quantum gravitodynamics<sup>4</sup> have been presented by the author. In quantum gravitodynamics, the author develops an approach to understanding the response of a lepton to a weak (linearized) gravitational field by making use of the vector representation of the linearized gravitational field<sup>5</sup>. In the covariant perturbation theory approach, the quantum theory of gravity is rendered finite by making use of a Euclidean, retarded, graviton propagator given by:

$$(\delta\mu\rho\delta\nu\sigma + \delta\mu\sigma\delta\nu\rho - \delta\mu\nu\delta\rho\sigma) \frac{e^{-k^2/m^2}}{k^2} \quad (1.1)$$

where  $\frac{1}{m}$  is the graviton Compton wavelength given by  $6.7 \times 10^{-4}R$  where  $R = c/H$  is the ‘‘Hubble radius’’ of the universe and  $H$  is the Hubble constant. The graviton propagator is defined in the linear approximation since the notion of mass and spin of a field requires the presence of a flat background metric  $\eta_{\mu\nu}$  which one does not have in the full theory. The full theory of general relativity may then be viewed as that of a graviton field which undergoes a

nonlinear self-interaction. The propagator in Eq. (1.1) will render such a full theory finite to all orders. It is the discovery of this propagator which motivates us to study the possibility of unifying the graviton field with the existing electroweak theory. It is known that linearized gravity predicts that the motion of masses produces magnetic gravitational effects very similar to electromagnetism<sup>5</sup>. The effective interaction between the electron and the graviton field can be understood in the vector representation where we make use of a propagator with the functional dependence given in Eq. (1.1) but with suitable vector indices. The author has calculated the order  $\alpha$  correction to the magnetic gravitational moment by using such a propagator<sup>4</sup>. Therefore, we are motivated to propose a gauge group  $SU(3) \times SU(2) \times U(1) \times U(1)$  for the unified field theory which incorporates the strong force, the weak and electromagnetic interaction, and the graviton field. In this paper, we focus on the  $SU(2) \times U(1) \times U(1)$  sector of the gauge theory. The feasibility of such a gauge structure and its implications for the existence of massive vector bosons, the  $Y^\pm$  and the  $X^0$ , and the determination of their mass spectrum are studied in this paper. We also predict a modification to the fine structure constant under unified field conditions. Furthermore, the consequences of the extended object formulation for the gauge hierarchy problem are examined.

## II. $SU(2) \times U(1) \times U(1)$

Let us consider the electronic-type lepton fields which consist of only the left- and right-handed parts of the electron field  $e$ :

$$e_L = \frac{1}{2}(1 + \gamma_5)e, \quad e_R = \frac{1}{2}(1 - \gamma_5)e \quad (2.1)$$

and a purely left-handed electron-neutrino field  $\nu_{eL}$ :

$$\gamma_5 \nu_{eL} = \nu_{eL}. \quad (2.2)$$

In any representation of the gauge group, the fields must all have the same Lorentz transformation properties, so the representations of the gauge group must divide into a left-handed doublet  $(\nu_{eL}, e_L)$  and a right handed singlet  $e_R$ . Thus, the largest possible gauge group is then

$$SU(2) \times U(1) \times U(1) \quad (2.3)$$

under which the fields transform as

$$\delta \begin{pmatrix} \nu_e \\ e \end{pmatrix} = i [\vec{\epsilon} \cdot \vec{t} + \epsilon_L t_L + \epsilon_R t_R] \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad (2.4)$$

where the generators are

$$\vec{t} = \frac{g}{4}(1 + \gamma_5) \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}, \quad (2.5)$$

$$t_L \propto (1 + \gamma_5) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (2.6)$$

$$t_R \propto (1 - \gamma_5) \quad (2.7)$$

$$(2.8)$$

with  $g$  an unspecified constant. It will be convenient instead of  $t_L$  and  $t_R$  to consider the generators

$$y = g' \left[ \frac{(1 + \gamma_5)}{4} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{(1 - \gamma_5)}{2} \right] \quad (2.9)$$

and

$$n_e = g'' \left[ \frac{(1 + \gamma_5)}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{(1 - \gamma_5)}{2} \right], \quad (2.10)$$

where  $g'$  and  $g''$  are unspecified constants like  $g$ . The generator  $y$  (the hypercharge) appears along with  $t_3$  (the isospin operator) in a linear combination to define the charge  $q$  of the pair  $(\nu_{eL}, e_L)$ :

$$q = e \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} = \frac{e}{g} t_3 - \frac{e}{g'} y. \quad (2.11)$$

Also,  $n_e$  is the electron-type lepton number and it defines the mass of the left-handed pair  $(\nu_{eL}, e_L)$  and the right-handed singlet  $e_R$ :

$$m_{ab} = m \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{m}{g''} n_e \quad (2.12)$$

where  $m$  is the electron mass. Thus, the charge couples to the electromagnetic field and the mass (in geometrized units) couples to the weak gravitational field. We want to include

charge changing weak interactions (like beta decay), electromagnetism, and the graviton field in our theory, so we will assume there are gauge fields  $\vec{A}^\mu$ ,  $B^\mu$ , and  $C^\mu$  coupled to  $\vec{t}$ ,  $y$ , and  $n_e$  respectively. Before we include the graviton field in our theory we must ensure that it satisfies the stringent limits on long range forces that would be produced by a massless gauge field coupled to  $n_e$ <sup>6</sup>. Since the gravitational interaction is much weaker than the weak or electromagnetic interactions we are free to include a gauge field  $C^\mu$  with strength  $g''$  coupled to  $n_e$ . The gauge group is then

$$G = SU(2)_L \times U(1) \times U(1) \quad (2.13)$$

where the generators  $\vec{t}$ ,  $y$ , and  $n_e$  are given by Eq. (2.5), Eq. (2.9), and Eq. (2.10) respectively. The most general gauge- invariant and renormalizable Lagrangian that involves gauge-fields and electronic leptons is

$$\begin{aligned} \mathcal{L}_{YM} + \mathcal{L}_{LG} + \mathcal{L}_e = & -\frac{1}{4} \left( \partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g \vec{A}_\mu \times \vec{A}_\nu \right)^2 \\ & -\frac{1}{4} \left( \partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu \right)^2 - \frac{1}{4} \left( \partial_\mu \vec{C}_\nu - \partial_\nu \vec{C}_\mu \right)^2 \\ & -\bar{l} \left( \not{\partial} - i\vec{A} \cdot \vec{t} - i\mathcal{B}y - i\mathcal{C}n_e \right) l. \end{aligned} \quad (2.14)$$

The coupling constants  $g$  and  $g'$  are to be adjusted so that the gauge fields  $\vec{A}^\mu$ ,  $B^\mu$ , and  $C^\mu$  coupled to these generators are canonically normalized. Now, of these five gauge fields coupled to  $\vec{t}$ ,  $y$ , and  $n_e$ , only two linear combinations, the electromagnetic field  $A_\mu$ , and the graviton field (vector representation)  $A_\mu^G$  are actually massless. We therefore must assume that  $SU(2)_L \times U(1) \times U(1)$  is spontaneously broken into  $U(1)_{em} \times U(1)_{gravity}$  with generators given by the hypercharge  $y$  and the electron-type lepton number  $n_e$ . The details of the symmetry-breaking mechanism will be considered a little later. However, whatever this mechanism may be, we know that the canonically normalized vector fields corresponding to particles of spin one and definite mass consist of one field of charge  $+e$  with mass  $m_Y$

$$Y^\mu = \frac{1}{\sqrt{2}} (A_1^\mu + iA_2^\mu) \quad (2.15)$$

and another of charge  $-e$  and the same mass

$$Y^{\mu*} = \frac{1}{\sqrt{2}}(A_1^\mu - iA_2^\mu) \quad (2.16)$$

and three electrically neutral fields of mass  $m_X$ , zero, and zero respectively given by orthonormal linear combinations of  $A_3^\mu$ ,  $B^\mu$ , and  $C^\mu$ :

$$X^\mu = \cos \phi A_3^\mu + \sin \phi B^\mu \quad (2.17)$$

$$A^\mu = -\cos \theta \sin \phi A_3^\mu + \cos \theta \cos \phi B^\mu + \sin \theta C^\mu \quad (2.18)$$

$$A_G^\mu = \sin \theta \sin \phi A_3^\mu - \sin \theta \cos \phi B^\mu + \cos \theta C^\mu \quad (2.19)$$

where  $\phi$  is the electroweak mixing angle (the Weinberg angle) and  $\theta$  is the gravitoweak mixing angle. These linear combinations employ the Euler angles for a transformation from space axes to body coordinates with the third rotation set to zero. In this theory, the third rotation is set to zero because both the electromagnetic and graviton fields are massless  $U(1)$  gauge fields and two  $U(1)$ 's are independent of each other. Hence, the mixing angle between the electromagnetic and graviton fields (the electrogravity angle) is zero. The electromagnetic field mixes with the weak interaction via the Weinberg angle and the weak interaction in turn mixes with the graviton field via the gravitoweak mixing angle. By making use of the inverse transformation back to space axes we have:

$$A_3^\mu = \cos \phi X^\mu - \cos \theta \sin \phi A^\mu + \sin \theta \sin \phi A_G^\mu \quad (2.20)$$

$$B^\mu = \sin \phi X^\mu + \cos \theta \cos \phi A^\mu - \sin \theta \cos \phi A_G^\mu \quad (2.21)$$

$$C^\mu = \sin \theta A^\mu + \cos \theta A_G^\mu \quad (2.22)$$

In the limit as the gravitoweak angle  $\theta$  goes to zero, we recover the linear combinations necessary to generate the electroweak mass spectrum<sup>7</sup>. In the above linear combinations we observe that the massive fields  $Y_\pm^\mu$  and  $X^\mu$  are specified entirely in terms of the gauge fields  $\vec{A}^\mu$  and  $B^\mu$ . Since the electrogravity mixing angle is zero, spontaneous symmetry breaking, which generates the vector meson term, occurs only in the electroweak sector of the theory. However, the coupling constants  $g$  and  $g'$  of the electroweak sector are specified in terms of the coupling constant  $g''$  of the gravity sector as shown below. Thus, the spontaneous symmetry breaking

of  $SU(2)_L \times U(1) \times U(1)$  into  $U(1) \times U(1)$  will generate two massless particles, namely, the photon and the graviton. Now, the generators of the unbroken symmetries, which are here electromagnetic and gravitodynamic gauge invariance are given by a linear combination of generators in which the coefficients are the same as the coefficients of the canonically normalized gauge fields coupled to these generators<sup>7</sup>. Inspecting Eqs.(2.20) shows that

$$q = -\cos\theta \sin\phi t_3 + \cos\theta \cos\phi y, \quad (2.23)$$

$$m_{ab} = \cos\theta n_e. \quad (2.24)$$

Comparing this with Eq.(2.11) and Eq.(2.12) gives then

$$g = \frac{-e}{\cos\theta \sin\phi}, \quad g' = \frac{-e}{\cos\theta \cos\phi}, \quad g'' = \frac{m}{\cos\theta}. \quad (2.25)$$

To complete the theory, we must now make some assumption about the mechanism of symmetry breaking. This mechanism must give masses not only to the  $Y^\pm$  and  $X^0$ , but to the electron as well. Thus, we assume a ‘Yukawa’ coupling

$$\mathcal{L}_\phi = -G_e \overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + H.c., \quad (2.26)$$

where  $(\phi^+, \phi^0)$  is a doublet on which the  $SU(2)_L \times U(1)$  generators are represented by the matrices:

$$\vec{t}^{(\phi)} = \frac{g}{2} \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}, \quad (2.27)$$

$$y^{(\phi)} = \frac{-g'}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.28)$$

so that the charge matrix is

$$q = e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{e}{g} t_3^{(\phi)} - \frac{e}{g'} y^{(\phi)}. \quad (2.29)$$

The most general form of the gauge-invariant term involving scalar and gauge fields consistent with the  $SU(2)_L \times U(1)$  sector of the theory is:

$$\mathcal{L}_\phi = \frac{-1}{2} \left| \left( \partial_\mu - i \vec{A} \cdot \vec{t}^{(\phi)} - i B_\mu y^{(\phi)} \right) \phi \right|^2 - \frac{\mu^2}{2} \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2 \quad (2.30)$$

where  $\lambda > 0$  and

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \quad (2.31)$$

For  $\mu^2 < 0$ , there is a tree-approximation vacuum expectation value at the stationary point of the Lagrangian

$$\langle \phi \rangle \langle \phi^\dagger \rangle = v^2 = |\mu^2|/\lambda \quad (2.32)$$

In unitarity gauge the vacuum expectation values of the components of  $\phi$  are

$$\langle \phi^+ \rangle = 0, \quad \langle \phi^0 \rangle = v > 0. \quad (2.33)$$

The scalar Lagrangian Eq. (2.30) then yields a vector meson mass term of the form

$$\frac{-v^2 g^2}{4} Y_\mu^\dagger Y_\mu - \frac{v^2}{8} (g^2 + g'^2) X_\mu X^\mu, \quad (2.34)$$

where

$$\frac{g}{g''} = \frac{-e/m}{\sin \phi}, \quad (2.35)$$

$$\frac{g'}{g''} = \frac{-e/m}{\cos \phi}, \quad (2.36)$$

$$g'' = \frac{m}{\cos \theta}. \quad (2.37)$$

Here,  $\phi$  is the Weinberg angle and  $\theta$  is the gravitoweak mixing angle. We see that the photon mass is zero corresponding to an unbroken gauge symmetry  $U(1)_{em}$  and the graviton mass is also zero corresponding to another unbroken gauge symmetry  $U(1)_{gravity}$  while the  $Y^\pm$  and  $X^0$  have the masses

$$m_Y = \frac{v|g|}{2}, \quad m_X = \frac{\sqrt{g^2 + g'^2}}{2}. \quad (2.38)$$

Now, consider the relation<sup>7</sup>

$$g^2/m_Y^2 = 4\sqrt{2}G_F \quad (2.39)$$

where  $G_F = 1.16639(2) \times 10^{-5} \text{GeV}^{-2}$  is the Fermi constant. This relation is obtained by comparing the effective interaction between low energy,  $e$ -type and  $\mu$ -type leptons with the

effective 'V-A' theory which is known to give a good description of muon decay. This allows an immediate determination of the vacuum expectation value as

$$v = \frac{2m_Y}{g} = 247\text{GeV}. \quad (2.40)$$

By making use of the known value of the electroweak mixing angle given by  $\sin^2 \phi = 0.23$ , we can determine the masses of  $Y^\pm$  and  $X^0$  in terms of the gravitoweak mixing angle as:

$$m_Y = \frac{e_\mu v}{2|\cos \theta| |\sin \phi|} = \frac{80.2\text{GeV}}{|\cos \theta|} \quad (2.41)$$

$$m_X = \frac{e_\mu v}{2|\cos \theta| |\sin 2\phi|} = \frac{91.3\text{GeV}}{|\cos \theta|} \quad (2.42)$$

where  $e_\mu$  is the electric charge defined at a sliding scale  $\mu$  comparable to the energies of interest. We observe that as  $\theta \rightarrow 0$  we regain the  $W$  and  $Z$  boson masses which is a result we expect. Thus, a unified field theory predicts the existence of massive vector bosons  $Y^\pm$  and  $X^0$  with the mass spectrum given in Eqs.(2.41)-(2.42). If we express the covariant derivative in Eq. (2.30) in terms of the mass eigenstate fields  $Y_\mu^\pm$ ,  $X^\mu$ , and  $A_\mu$  we find that the coefficient of the electromagnetic interaction is not the electron charge  $e$ , but rather the effective electron charge  $e'$

$$e' = \frac{e}{|\cos \theta|} = \frac{gg'}{\sqrt{g^2 + g'^2}}. \quad (2.43)$$

We observe that  $e' \geq e$  with equality being achieved when the gravitoweak mixing angle  $\theta$  is zero. The mixing between the weak interaction and the graviton field causes an increase in the electromagnetic coupling strength. This is because the electromagnetic coupling is a function of the gauge couplings  $g$  and  $g'$  which have a dependence on  $\theta$ . If  $\alpha_G$  is the fine structure constant of an electron in the unified field, then we have:

$$\frac{\alpha_G}{\alpha} = \frac{1}{\cos^2 \theta} \quad (2.44)$$

implying that the fine structure constant suffers a modification. This would mean that if we were to measure the Lamb shift under unification conditions, the correction to the  $g$ -factor of the electron would be

$$a_e = \frac{\alpha_G}{2\pi} = \frac{0.0011597}{\cos^2 \theta}. \quad (2.45)$$



### III. THE GAUGE HIERARCHY PROBLEM

We begin with the reasonable observation that if  $SU(2) \times U(1)$  is broken by the vacuum expectation value of an elementary scalar field, then that scalar field should be part of the grand unification. In order to produce a vacuum expectation value of the right size to give the observed  $W$  and  $Z$  boson masses, the Higgs scalar field must obtain a negative mass term of the size<sup>8</sup>

$$-\mu^2 \sim -(100\text{GeV})^2. \quad (3.1)$$

Now, the mass term can be expressed in terms of the vacuum expectation value  $v$  as

$$|\mu^2| = \lambda v^2 \quad (3.2)$$

where  $\lambda$  is the renormalizable coupling in  $(\phi^\dagger\phi)^2$  charged scalar field theory. Therefore, the  $(mass)^2$  receives additive renormalizations. In a theory with a cutoff scale  $\Lambda$ ,  $\mu^2$  can be much smaller than  $\Lambda^2$  only if the bare mass of the scalar field is of the order  $-\Lambda^2$  and this value is canceled down to  $-\mu^2$  by radiative corrections. If our theory of nature contains very large scales of grand unification, then the appropriate value for  $\Lambda$  is  $10^{16}$  GeV or larger and it would require bizarre cancellations in the renormalized value of  $\mu^2$ . Thus, the Higgs boson mass is very small compared to the grand unification scale. It is a mystery as to why the  $(mass)^2$  of the Higgs boson has a value 28 orders of magnitude or more below its natural value and this question is referred to as the *gauge hierarchy problem*. However, at grand unification energy scales the contributions of hitherto nonrenormalizable terms such as the Pauli term become significant<sup>3</sup>. The description of quantum electrodynamics with the Pauli term necessitates the introduction of the quantum field theory of extended objects in which the finite extent of a particle defined via its Compton wavelength is incorporated into the field structure and leads to a finite interaction. Since hitherto nonrenormalizable terms become important at grand unification scales, it would be more correct if we consider  $SU(2) \times U(1)$  to be broken by the vacuum expectation value of a hitherto nonrenormalizable  $(\phi^\dagger\phi)^3$  scalar field which can be rendered finite in the extended

object formulation<sup>2</sup>. The coupling  $\lambda$  now becomes a finite coupling and the  $(mass)^2$  does not receive additive renormalizations. Consider the potential

$$V(\phi) = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^3 \quad (3.3)$$

which has a tree-approximation vacuum expectation value at

$$\langle\phi\rangle\langle\phi^\dagger\rangle = (|\mu^2|/\lambda)^{\frac{1}{2}} \quad (3.4)$$

implying that

$$|\mu^2| = \lambda v^4 \quad (3.5)$$

where  $\lambda$  is now a finite coupling. Therefore, we can now expect the Higgs boson mass to be of the order of 100 GeV without any conceptual difficulty.

#### IV. CONCLUSION

The general gauge group  $SU(3) \times SU(2) \times U(1) \times U(1)$  appears to describe the four known interactions in a consistent fashion. We are able to predict the existence of gauge bosons  $Y^\pm$  and  $X^0$  for the  $SU(2) \times U(1) \times U(1)$  sector of this unified theory and determine mass spectrum of the gauge bosons. We have also shown that the fine structure constant is modified under unified field conditions. In addition, a possible resolution of the gauge hierarchy problem has been discussed. The results of this paper need to be subjected to experimental tests.

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